Proposal of Robust Design Method for Diverse Conditions in Consideration of Cost

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Abstract: Robust design ensures products have a strong performance under diverse conditions. A previous study proposed a robust design method for diverse conditions, which allows robustness to be measured in problems where the objective characteristics have non-normal distributions, and in problems where the control factors have adjustable ranges. In the latter problem, a user can set the value of the control factors, such as an adjustable mechanism design, which strives to increase robustness. However, because increasing the adjustable range or number of mechanisms increases production cost, adjustable mechanism designs should be evaluated with respect to the trade-off between robustness and production cost. Herein we propose a method employing a quality loss function, which is the relationship between the quality and cost. In this method, the bi-objective optimization problem of the robustness and cost is treated as a cost minimization problem. Furthermore, we apply this method to an example of a public vehicle seat.

Key words: Design Methodology, Robust Design, Production Cost, Seat Design.

1. Introduction

A product is designed with consideration to user needs and usage environment. However, in recent years, user needs have diversified and the market for a product has become globalized. Additionally, the conditions, which are the design object, user features, and the environments the object will be used, have diversified. Hence, the designer should consider the diverse conditions.

A robust design method can be applied to problems of variations, which are machining and assembly errors. Matsuoka et al. [1] developed this method to ensure the function of a product is robust over diverse conditions. This method is applicable to two design problems: problems where the design solution is a fixed value set by the designer and problems where the solution has multiple values so that users can adjust the value. For the former problem, an index to evaluate this distribution has been proposed because the objective characteristic has a non-normal distribution due to diverse conditions. For the latter, taking multiple values of the design solution has been purposed to increase robustness, and is achieved by employing an adjustable mechanism, such as a headrest in a car. To derive these values, an index to evaluate robustness when a design parameter has an adjustable range has been proposed.

However, because increasing the adjustable range or the number of mechanisms increases the production cost, an adjustable mechanism design should be evaluated in consideration of the trade-off between robustness and production cost. Herein, we focus on a quality loss function, which is the relationship between quality and cost. A method to evaluate both robustness and cost based on this loss function is proposed. Furthermore, we apply the proposed method to a public vehicle seat.

2. Robust design method for diverse conditions

We use the robust design method for diverse conditions as a method to evaluate robustness. This method consists of two indices, which evaluate the probability density function of the objective characteristic: the robustness index R, which is applicable to design problems with a non-normal distribution with multiple peaks, and the robustness index for adjustable control factor R_A, which is applicable to design problems with the control factor possessing an adjustable range, i.e. design problems with an adjustable mechanism, where control factor is those that a designer can control.

2.1. Robustness index

The robustness index R is the feasibility of the objective characteristic value being within a tolerance, as shown in Fig. 1. By integrating the probability density function of the objective characteristic, R can be represented as follows:.

$$R = \int_{y_l}^{y_u} p(y)dy = \int_{y_l \le f(\boldsymbol{x},\boldsymbol{z}) \le y_u} p(\boldsymbol{x},\boldsymbol{z})d\boldsymbol{x}d\boldsymbol{z} \tag{1}$$

where y is the objective characteristic, $p(y)$ is the probability density function of y, y_l and y_u are the lower and upper tolerance limits, respectively. x represents the control factors. z denotes noise factors, which cannot be controlled by the designer. $f(x, z)$ is the objective function of x and z. R is an index to evaluate the robustness of the objective characteristic with a non-normal distribution.

In calculating R, the double integral on the right side of Eq. (1) deals as the iterated integral. However, the function of many variables, which consist of x and z, does not always have an implicit function in the integral range of R. Moreover, when the objective function stochastically fluctuates or is discontinuous, calculating the integral range is difficult. Therefore, R is calculated using the Monte Carlo method. First, a random number of variation factors is generated based on the probability density function of their variation. Second, the objective characteristic values of each random number are calculated. Finally, by assessing whether the values are within a tolerance, R is calculated as follows:

$$R = \frac{1}{n}\sum_{i=1}^{n} M_i \qquad \left(M_i = \begin{cases} 1 & (y_l \le f(\boldsymbol{x}_i,\boldsymbol{z}_i) \le y_u) \\ 0 & (otherwise) \end{cases} \right) \tag{2}$$

where n is the number of samples generated for a random value of variation factors. Calculating R requires considerable time, because to improve the precision of the probability density function of y, n becomes larger.

2.2. Robustness index for adjustable control factor

An adjustable control factor t is a control factor with an adjustable range, which can be altered by users or designers. As shown in Fig. 2, the robustness index for adjustable control factor R_A is the ratio of the sum of sets of the probability of the objective characteristic value being within the tolerance for each value of t:

$$R_A = P\left[\cup \left\{ C_{t_i}(\boldsymbol{x},\boldsymbol{z}) \middle| y_l \le f(\boldsymbol{x},\boldsymbol{z},t_i) \le y_u \right\}\right] \qquad \left(t \in S \middle| \left| t - t^* \right| \le \Delta \right) \tag{3}$$

where $P[A]$ is the feasibility of event A, i is the level of the adjustable control factor, t^* is the initial value of t, and Δt is the adjustable range of t. The robustness can be evaluated using R_A, not only in the case where the control factor has an adjustable range, but also where the solution using R provides inadequate robustness and an additional design with an adjustable mechanism is required. Similar to R, R_A is calculated using the Monte Carlo method, which requires vast computation time, but the calculation method can be divided into two procedures by the derivative of the objective function.

First, when the derivative of the objective function is a positive or negative definite, the magnitude of the relationship of the values of the objective characteristic is inferred from that of the values of the control factor as shown in Fig. 3(a). Therefore, by evaluating percentiles of the control and noise factors at the upper and lower percentage of the objective characteristic, an adjustable range can be calculated as follows:

$$R_A = 1 - \frac{1}{s}\sum_{i=1}^{s} M_i \qquad \left(M_i = \begin{cases} 1(\{f(\boldsymbol{x}_i, \boldsymbol{z}_i, t_{max}) < y_l\}) \\ 1(\{f(\boldsymbol{x}_i, \boldsymbol{z}_i, t_{min}) > y_u\}) \\ 0(otherwise) \end{cases} \right) \tag{4}$$

Second, in the case where the derivative of the objective function is neither a positive nor negative definite, the objective function may not be monotonic, as shown in Fig. 3(b). Because the magnitude of the relationship between two arbitrary points of the adjustable factor may be reversed, it is necessary to examine all values of the adjustable control factor, control factors, and noise factors. Therefore, an adjustable range can be calculated as follows:

$$R_A = \frac{1}{s}\sum_{i=1}^{s} M_i \qquad \left(M_i = \begin{cases} 1 & \left(\exists t \in S \,\middle|\, |t - t^*| \le \Delta ; y_l \le f(\boldsymbol{x}_i, \boldsymbol{z}_i, t) \le y_u \right) \\ 0 & (otherwise) \end{cases} \right) \tag{5}$$

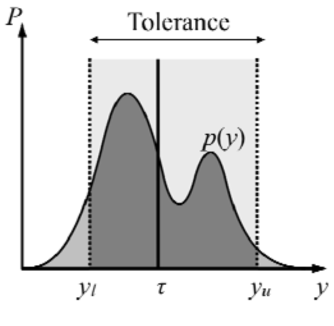


Fig. 1 Concept of the robustness index

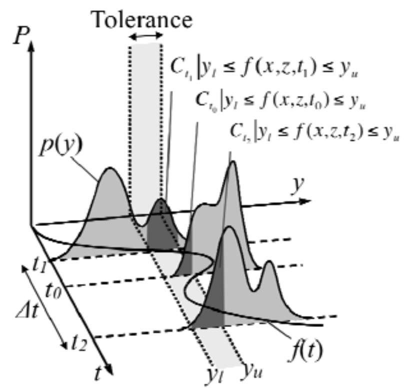
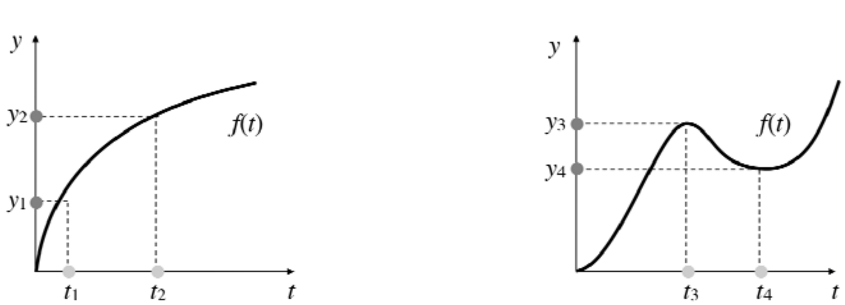


Fig. 2 Concept of the robustness index for adjustable control factor



(a) Positive definite (b) Not positive definite

Fig. 3 Derivative of the objective function

3. Robust design method in consideration of cost

Products must be evaluated in terms of cost and quality. Therefore, we developed a robust design method for diverse conditions to consider cost using a quality loss function. In this section, $R_{A(\Delta t=0)}$ represents R, because R_A is equal to R when Δt is zero.

3.1. Quality loss function

Dr. Taguchi, who first advocated the robust design method, proposed a quality loss function, which is used to determine the tolerance and safety factor [2]. This function converts the quality of a product into a cost, assuming that the difference between the actual quality and ideal quality is regarded as a financial loss. The function has three forms for the objective characteristics: nominal-is-best type, smaller-is-best type, and larger-is-best type. As an example, the nominal-is-best type problem is illustrated. In a nominal-is-best type problem, the bigger the gap between objective characteristic y and the target value m is, the greater the financial loss $L(y)$. However, to accurately estimate $L(y)$, a wide-scale social investigation is necessary. Therefore, the quality loss function is approximated by a Taylor series expansion around $y=m$ as follows:

$$\begin{aligned} L(y) &= L(m+y-m) \\ &= L(m) + \frac{L'(m)}{1!}(y-m) + \frac{L''(m)}{2!}(y-m)^2 + \cdots \end{aligned} \tag{6}$$

In this equation, the third order and higher order terms are omissible. $L(m)=0$ and $L'(m)=0$ are derived from the condition that the loss at the target value m is at a minimum, and the loss increases as y varies from m. Thus, the quality loss function can be rewritten as:

$$L(y) = k(y-m)^2 \tag{7}$$

where k is a constant consisting A and Δ. A is the limit where the product does not work and Δ is the cost at the limit, as shown in Fig. 4. In Eq. (7), $L(y)$ is proportional to the square of the difference between y and m. This is the reason for the assumption that the square of the error of the quality is proportional to the financial cost.

3.2. Production cost

Production cost is the cost incurred before shipping a product, and includes costs such as material cost and assembly cost. As shown in Fig. 5, the relationship between the production cost and R_A can be divided into two categories based on whether the mechanism is adjustable. First, when the adjustable mechanism is not set, the production cost is a constant value $L_p(0)$, and is independent of the magnitude of robustness. As an example, the seat height design in a public seat is shown. In an unadjustable mechanism, the height where $R_{A(\Delta t=0)}$ is the maximum value (R_u) is chosen as a design solution (Fig. 6(a)). Thus, materials and processing methods barely change, and the production cost does not increase. However, if an adjustable mechanism is designed, the production cost $\Delta_{(1)}$ increases with the increase in materials, complexity of the proceeding methods, and enlargement in the assembling process. In addition, as the adjustable range is extended, the number of the parts increases, and then cost $F_{(1)}(R_A)$ is required. Likewise, if the number of the adjustable mechanism shifts from one to two, $\Delta_{(2)}$ and $F_{(2)}(R_A)$ are also incurred. In the foregoing example of the public seat, if R_u is not satisfied with the required robustness, then an adjustable mechanism is needed. In the mechanism where the seat height can be changed as shown in Fig. 6(b), $\Delta_{(1)}$ is incurred because a gear or a drive shaft is attached. Additionally, when the adjustable range is extended, R_u changes into $R_{Au(1)}$, and the cost $F_{(1)}(R_A)$ increases due to the increased number of gear teeth, etc.

Therefore, the relationship between R_A and L_p is as follows:

$$L_{p(i)} = \begin{cases} \Delta_{(0)} \ (i=0) \\ L_{p(i-1)u} + \Delta_{(i)} + F_{(i)}(R_A) \ (1 \le i \le n) \end{cases} \qquad \left(dF_{(i)}(R_A)/dR_A \ge 0 \right) \tag{8}$$

where i is the number of the adjustable mechanisms, and n is the maximum number of i. The case where $i = 0$ is the unadjustable mechanism, and $\Delta_{(0)}$ represents the cost of the unadjustable mechanism.

3.3. Quality cost

Quality cost is the cost incurred after shipping the product, such as repair cost and loss for after-care. In this study, we applied the concept of the quality loss function: cost increases with a deviation from an ideal quality. Accordingly, the quality is regarded as the robustness of the objective characteristic, and the quality cost improves with the variation of the objective characteristic. Because the variation of the objective characteristic becomes wider as R_A decreases, the relationship between R_A and L_q is represented by the following equation and Fig. 7:

$$L_q = Q(R_A) \qquad \left(dQ(R_A)/dR_A < 0 \right) \tag{9}$$

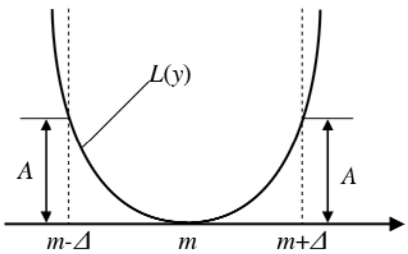


Fig. 4 Concept of the quality loss function

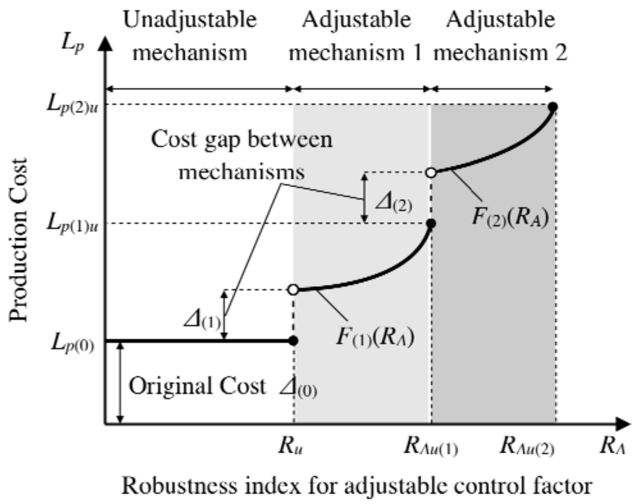
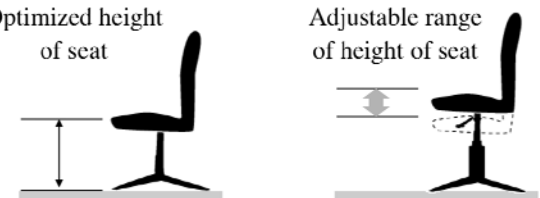


Fig. 5 Function of the production cost



(a) Unadjustable mechanism (b) Adjustable mechanism

Fig. 6 Public seat

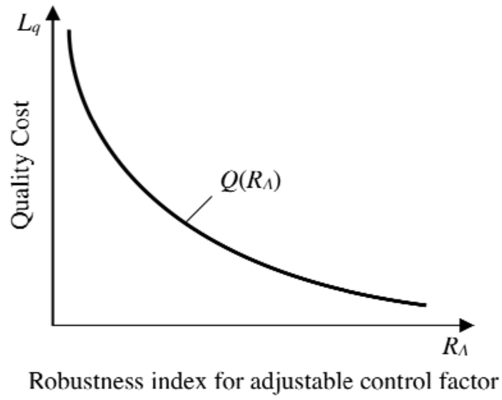


Fig. 7 Function of the quality cost

3.4. Cost minimization problem

A product can be evaluated as the sum of the product cost and the quality cost by converting the robustness to a quality cost. Hence, the bi-objective optimization problem of robustness and cost in the robust design method for diverse conditions is set as a cost minimization problem via the following equation:

$$\text{minimize} \quad L_{p(i)} + L_q \tag{10}$$

Additionally, Fig. 8 shows the design process considering robustness and cost.

4. Illustrative example

To demonstrate our method, we applied it to a public vehicle seat. The objective characteristic is a non-normal distribution, which is due to the stochastic fluctuation of multiple objective functions.

4.1. Problem description

Users with various physiques and sitting postures use public vehicle seats. However, traditional seat designs typically assume the average physique and posture. Thus, designing a seat that is robust for various physiques and postures, is desirable [3]. This study focuses on the hip-sliding force, which causes discomfort when sitting. The force is generated on the buttocks by the static instability of the upper and lower body masses. Accordingly, herein the design objective is to inhibit the hip-sliding force in various physiques and postures.

Specifically, the characteristics were modeled as follows. First, the objective characteristic was the hip-sliding force with a target value and tolerance of zero and from -10N to 20N, respectively [4]. Second, the control factor and the adjustable control factor were the cushion angle. Third, the noise factors were ascertained as the physique of the users and the sitting posture. The physiques were based on the actual measurements of Japanese citizens; the mean value and the standard deviation of the body height were 1.65 m and 0.08, respectively, and those of the body weight were 58.1 kg and 9.09 kg, respectively [5]. Additionally, three sitting postures were considered [6]: the standard sitting posture where the lumbar region is in contact with the seat back, stretched waist sitting posture where the waist is stretched and slid forward from the standard sitting posture, and the bent waist sitting posture where the waist is bent and slid forward from the standard sitting posture. The ratio of these sitting postures was 3:1:6. Finally, the objective function of each sitting posture was derived by modeling the sagittal plane of the human body and the seat (Fig. 9).

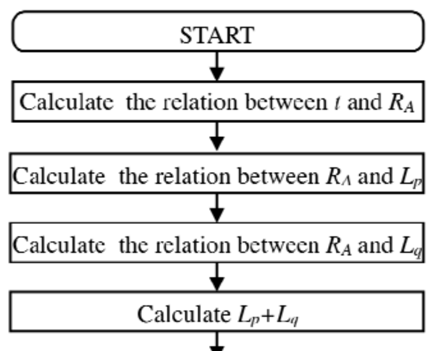


Fig. 8 Flow chart of the method

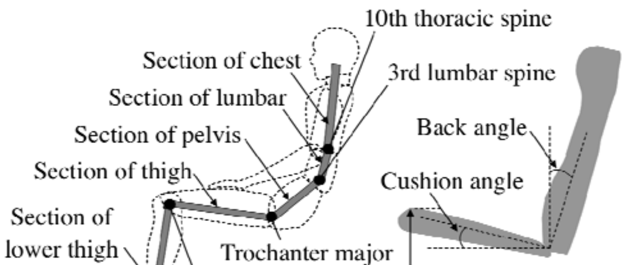


Fig. 9 Human and seat model

4.2. Cost setting

The cost of the seat was decided based on expert seat designers. The ratio of production cost of the unadjustable mechanism, that of the adjustable mechanism and the sales price was 10:1:20. Additionally, the production cost of the unadjustable mechanism per unit ($\Delta_{(0)}$) was 10,000 yen. Each cost was calculated as follows. The adjustable mechanism increased the production costs as the adjustable range increased. The ratio of $\Delta_{(1)}$ and $F_{(1)}(R_{Au(1)})$ was 1:1, and each cost was 500 yen.

$$L_{p(i)} = \begin{Bmatrix} 10000\ (i=0) \\ 10500 + F_{(1)}(R_A)\ (i=1) \end{Bmatrix} \qquad \left(dF_{(1)}(R_A)/dR_A \geq 0\right) \tag{11}$$

In contrast, the quality cost was ascertained as follows. The quality was defined as the loss when the users were uncomfortable when sitting. The percentage of seating comfort in the seat evaluation was 41% [7], and that of the hip-sliding force in the seating comfort was 56% [8]. Multiplying sales price by these percentages, the contribution to the hip-sliding force of the sales price was approximately 4500 yen. Therefore, the quality cost per unit was calculated by the following equation:

$$L_q = -4500R_A + 4500 \tag{12}$$

4.3. Results

The case where the mechanism was not adjustable was examined. The maximum value of $R_{A(\Delta t=0)}$ was calculated by varying the control factor x; $R_{A(\Delta t=0)}$ was 0.785 when θ_C was 18.4, indicating that robustness could be improved. Then we considered an adjustable mechanism design. The result was illustrated based on the flow of Fig. 8. First, the relationship between the adjustable control factor t and R_A was calculated. Increasing the adjustable range of t monotonically increased R_A, and it reached 1.00 in Δt=7.7. Second, the relationship between the production cost L_p and R_A was shown in Fig. 10. From Fig. 10 and Eq. (12) which was the relationship between the quality cost L_q and R_A, the cost of the sum of L_p and L_q was related to R_A as shown in Fig. 11. As a result, the minimum cost was 10,782 yen when R_A was 0.989 and the adjustable range of t was $17.9 \leq t \leq 19.2$. Hence, our method derived an optimal solution, which reflects the increased cost of an adjustable mechanism.

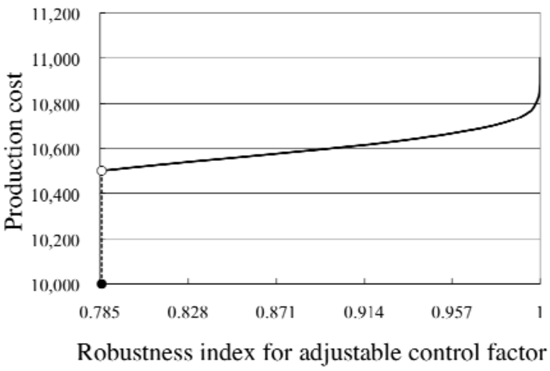


Fig. 10 Transition of the production cost

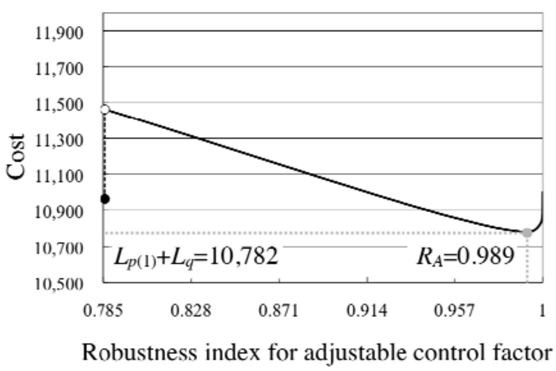


Fig. 11 Transition of cost

5. Conclusions

This study demonstrates a robust design method in consideration of cost. This method is developed from the robust design method for diverse conditions with respect to two costs: production cost, which is classified into two groups according to whether the mechanism is adjustable, and the quality cost, which represents the robustness as the financial loss. The relationship between cost and the indices of robustness is derived based on the quality loss function. From the relationship, the bi-objective optimization problem of the robustness and cost is treated as a cost minimization problem. Moreover, the effectiveness of the proposed method is illustrated by determining the appropriate cushion angle of a public vehicle seat. Consequently, the optimal solution, which considers both robustness and cost, was derived, indicating the potential of bi-objective optimization for both robustness and cost. Future research will address the development of the quality cost, such as the breakdown or product lifecycle, which are attributed to complex mechanisms.

6. References

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